UNIVERSITY OF TWENTE.

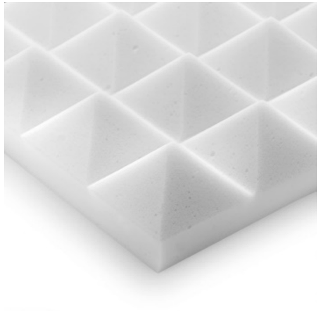
ABSORPTION OF SOUND

MASTER-CLASS ACOUSTIC FUNDAMENTALS

DUTCH ACOUSTICAL SOCIETY

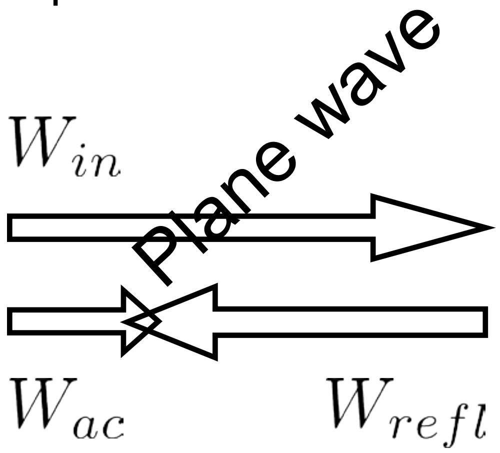
Y.H.WIJNANT

ABSORPTION OF SOUND



ABSORPTION OF SOUND

- Absorption

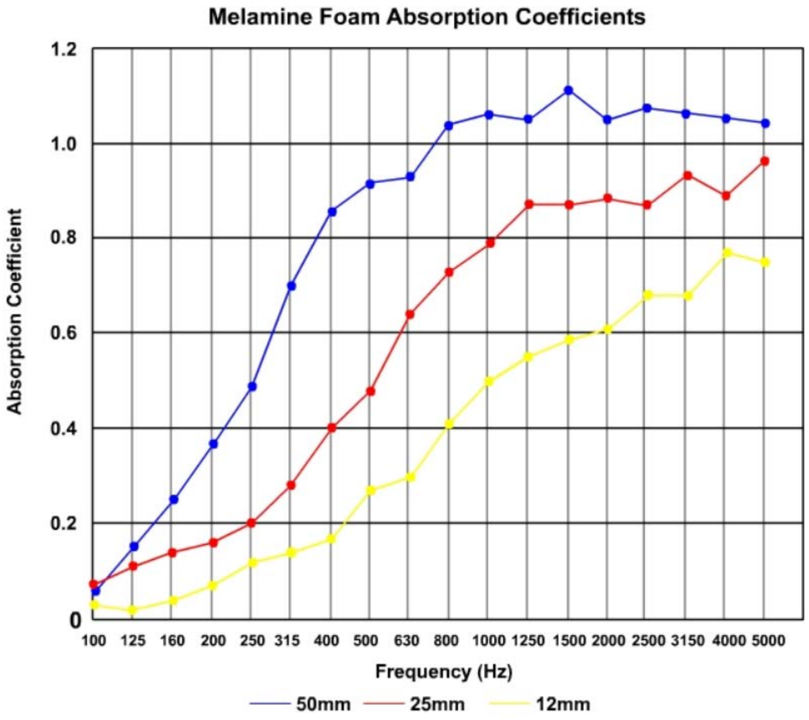


Active power

$$\alpha \equiv \frac{W_{ac}}{W_{in}}$$

Incident power

ABSORPTION OF SOUND



ABSORPTION OF SOUND

- Fundamentals
  - Wave equation - Helmholtz equation (complex notation)
  - Impedance
  - Intensity/Power
- Reflection / Transmission
- Absorption
- Demonstration

- Wave equation

$$\frac{\partial \delta\rho}{\partial t} + \rho_0 \frac{\partial u}{\partial x} = 0$$

$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0 \quad \longrightarrow \quad c_0^2 \frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 p}{\partial t^2} = 0$$

$$p = c_0^2 \delta\rho$$

$$c_0^2\frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 p}{\partial t^2} = 0$$

- Helmholtz equation (time harmonic solution)

$$p(x,t) = C(x)cos(\omega t - \phi)$$

$$p(x,t) = A(x)cos(\omega t) + B(x)sin(\omega t)$$

$$\left(c_0^2\frac{\partial^2 A(x)}{\partial x^2} + \omega^2 A(x)\right) cos(\omega t) + \left(c_0^2\frac{\partial^2 B(x)}{\partial x^2} + \omega^2 B(x)\right) sin(\omega t) = 0$$

$$\frac{\partial^2 A(x)}{\partial x^2} + \frac{\omega^2}{c_0^2}A(x) = 0 \qquad \frac{\partial^2 B(x)}{\partial x^2} + \frac{\omega^2}{c_0^2}B(x) = 0$$

- Helmholtz equation (time harmonic)

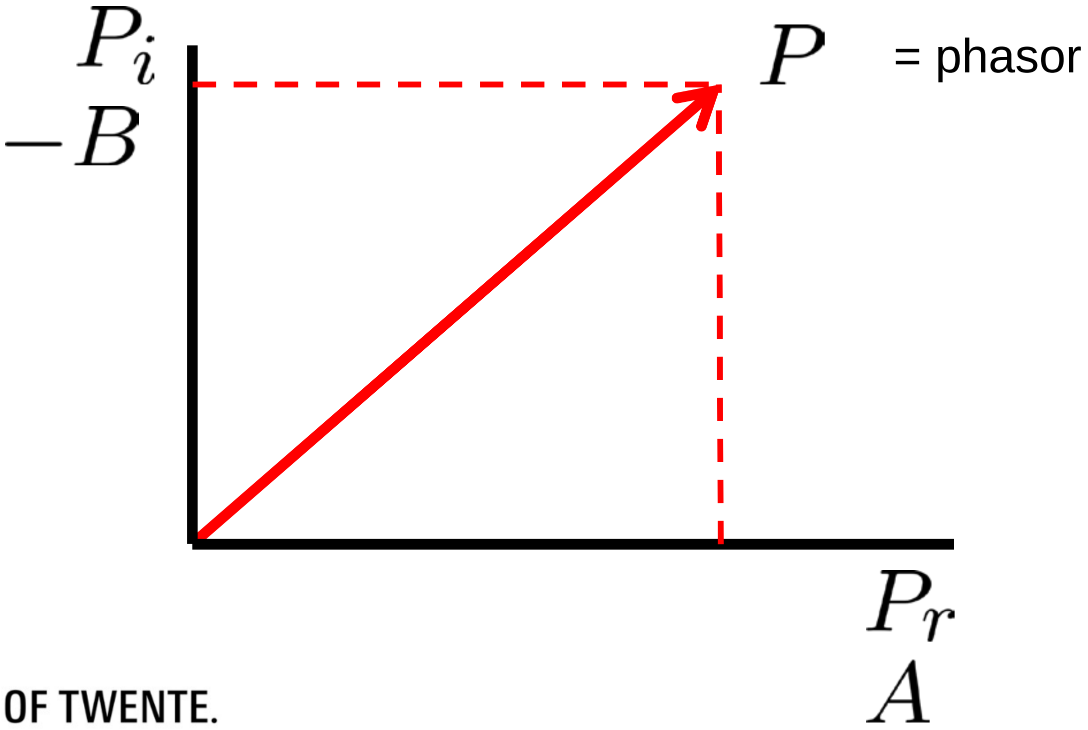
$$p(x,t) = A(x)cos(\omega t) + B(x)sin(\omega t)$$

  - equivalent (complex) notation

$$p(x,t) = \Re\left(P(x)e^{i\omega t}\right)$$

$$p(x,t) = \Re\{(P_r(x) + iP_i(x))(cos(\omega t) + isin(\omega t))\}$$

$$p(x,t) = P_r(x)cos(\omega t) - P_i(x)sin(\omega t)$$



P_i
$-B$
P
= phasor
P_r
A

$$c_0^2\frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 p}{\partial t^2} = 0$$

FUNDAMENTALS

- Helmholtz equation (time harmonic)

$$p(x,t) = \Re\left(P(x)e^{i\omega t}\right)$$

$$\frac{\partial^2 P(x)}{\partial x^2} + k^2 P(x) = 0$$

$$k = \frac{\omega^2}{c_0^2}$$

wave number [1/m]

$$\frac{\partial^2 P_r(x)}{\partial x^2} + \frac{\omega^2}{c_0^2} P_r(x) = 0$$

$$\frac{\partial^2 P_i(x)}{\partial x^2} + \frac{\omega^2}{c_0^2} P_i(x) = 0$$

FUNDAMENTALS

$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0$$

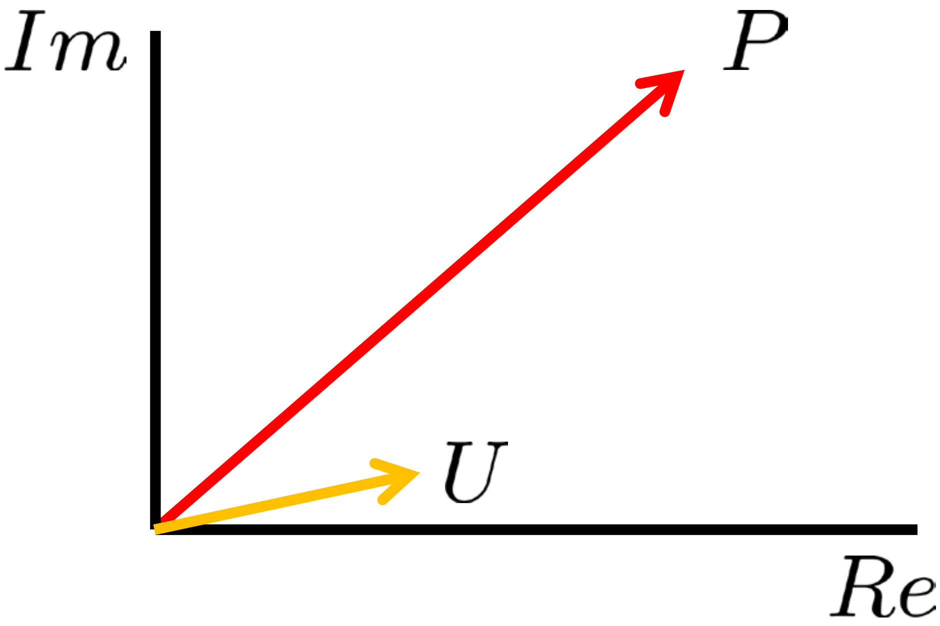
- Particle velocity

$$p(x,t) = \Re\left(P(x)e^{i\omega t}\right)$$

$$u(x,t) = \Re\left(U(x)e^{i\omega t}\right)$$

$$\left(\rho_0 i\omega U(x) + \frac{\partial P(x)}{\partial x}\right)e^{i\omega t} = 0$$

$$U(x) = \frac{-i}{\rho_0 \omega}\frac{\partial P(x)}{\partial x}$$



Im
P
U
Re

$$U(x) = \frac{-i}{\rho_0 \omega} \frac{\partial P(x)}{\partial x}$$

FUNDAMENTALS

- Plane wave solution

  - Purely propagating wave of amplitude A traveling in positive x-direction (no reflection)

$$P(x) = Ae^{-ikx}$$

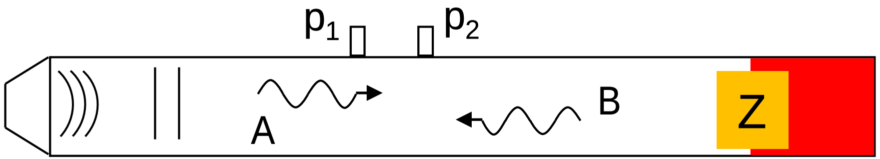
$$U(x) = \frac{1}{\rho_0 c_0} Ae^{-ikx}$$

$$\frac{P}{U} = \rho_0 c_0 = Z_0$$

- Characteristic specific acoustic impedance
  - = pressure over particle velocity ratio f
    purely propagating plan

If the impedance Z equals Z_0 then
the wave is a purely propagating plane wave!
If there is material present at that position
the plane wave is fully absorbed by that material!

  - [Rayl]
  - $Z_0 = 1.5\ 10^6$ [Rayl]



- ‘Incoming’ wave of amplitude A traveling in positive x-direction
- ‘Reflected’ wave of amplitude B traveling in the negative x-direction

$$P(x) = Ae^{-ikx} + Be^{ikx}$$

$$U(x) = \frac{A}{Z_0}e^{-ikx} + \frac{B}{-Z_0}e^{ikx}$$

$$\rightarrow \quad \frac{P(x)}{U(x)} \neq Z_0$$

- Specific acoustic impedance
  - = ratio (complex!) pressure over (complex!) particle velocity
  - Z = phasor

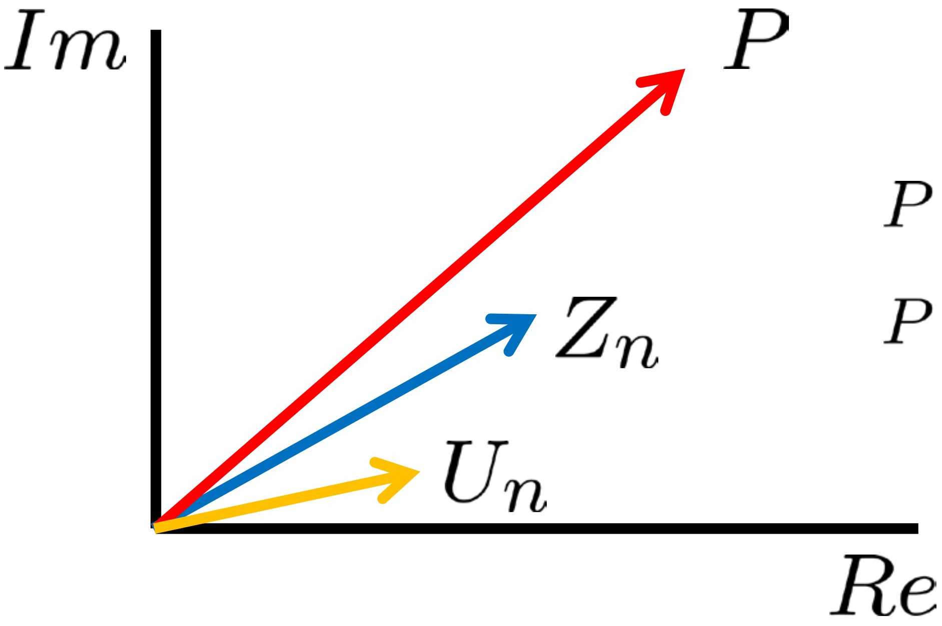
$$Z(x) = \frac{P(x)}{U(x)}$$

  - Note: only a frequency domain definition!

- Normal impedance

$$Z_n = \frac{P}{U_n}$$

FUNDAMENTALS



$$P = Z_n U_n$$

$$P = |Z_n||U_n|e^{i(\phi_Z + \phi_U)}$$

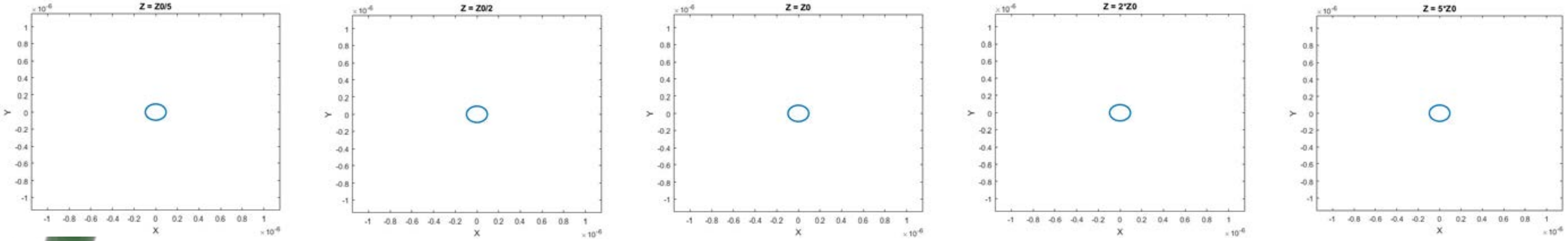
FUNDAMENTALS

- Relative specific acoustic impedance
  - relative to the characteristic impedance $Z_0 = \rho_0 c_0$

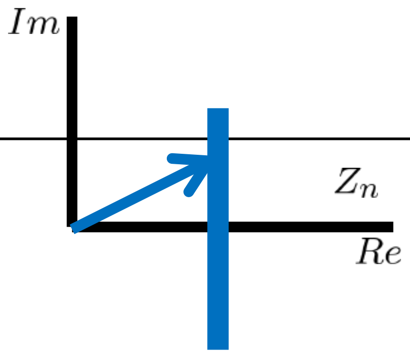
$$\zeta = \frac{Z}{\rho_0 c_0}$$

FUNDAMENTALS

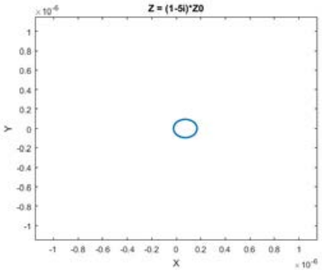
- Particle displacement / expansion
  - Changing ζ_r ($\zeta_i = 0$)



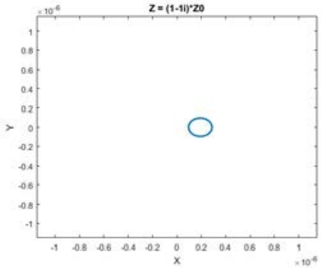
$\zeta_r = 1/5$ $\zeta_r = 1/2$ $\zeta_r = 1$ $\zeta_r = 2$ $\zeta_r = 5$



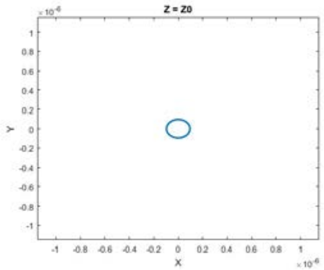
- Particle displacement / expansion
  - Changing ζ_i ($\zeta_r = 1$)



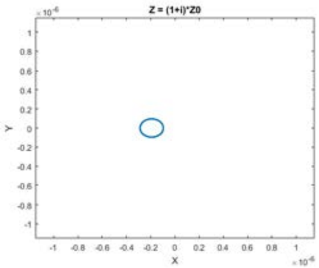
$\zeta_i = -5$



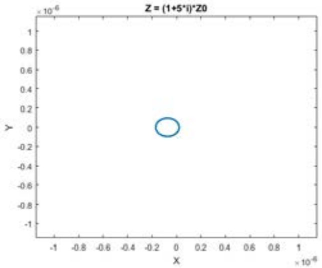
$\zeta_i = -1$



$\zeta_i = 0$



$\zeta_i = 1$



$\zeta_i = 5$

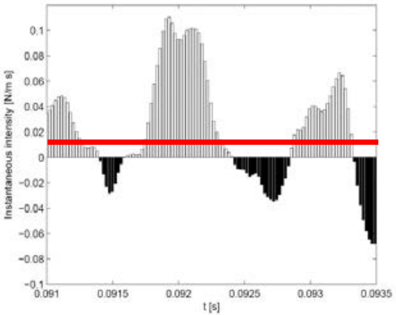
$$F\,dx = pS\,udt$$

FUNDAMENTALS

- Instantaneous acoustic intensity = energy flux [Watt/m^2]

$$i_n(x,t) = p(x,t)\,u_n(x,t)$$

- Active acoustic intensity = averaged over time



$$I_{ac} = \frac{1}{T}\int_0^T p(x,t)u_n(x,t)dt$$

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- Active intensity for harmonic signals averaged over one period

$$p(x,t) = P_r cos(\omega t) - P_i sin(\omega t)$$
$$u(x,t) = U_r cos(\omega t) - U_i sin(\omega t)$$

$$I_{ac} = \frac{1}{T}\int_0^T p(x,t)u_n(x,t)dt$$
$$T = \frac{2\pi}{\omega}$$

→

$$I_{ac} = \frac{1}{2}\Re\left(P\overline{U}\right)$$
$$I_{ac} = \frac{1}{2}U\overline{U}\,\Re(Z)$$
$$I_{ac} = \frac{1}{2}P\overline{P}\,\Re(1/\overline{Z})$$
$$\frac{1}{2}P\overline{P} = p_{rms}^2$$

FUNDAMENTALS

$$I_{ac} = \frac{1}{2}\Re\left(P\overline{U}\right)$$

- Intensity of the incident plane wave

$$P(x) = Ae^{-ikx}$$

$$U(x) = \frac{A}{Z_0}e^{-ikx}$$

$$\longrightarrow \quad I_{in} = \frac{A\overline{A}}{2\rho_0 c_0}$$

FUNDAMENTALS

$$I_{ac} = \frac{1}{2}\Re\left(P\overline{U}\right)$$

- Active intensity

  = intensity of the incident plane wave

  minus the intensity of the reflected wave

$$P(x) = Ae^{-ikx} + Be^{ikx}$$

$$U(x) = \frac{A}{Z_0}e^{-ikx} + \frac{B}{-Z_0}e^{ikx}$$

→

$$I_{ac} = \frac{A\overline{A}}{2\rho_0 c_0} - \frac{B\overline{B}}{2\rho_0 c_0}$$

$I_{ac} = I$ … efl

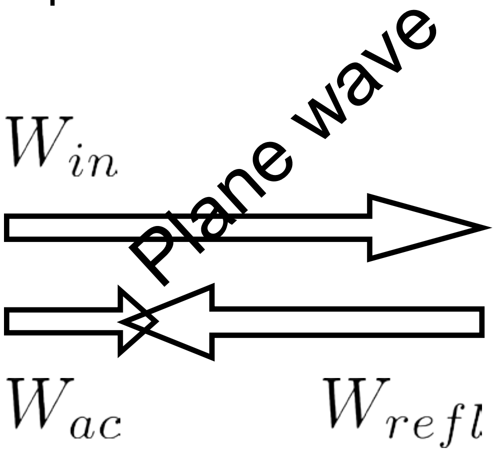
only true for 1D!

- Active power = spatial integration of the intensity [Watt]

$$W_{ac} = \int I_{ac} dS$$

ABSORPTION OF SOUND

- Absorption



Active power

$$\alpha \equiv \frac{W_{ac}}{W_{in}}$$

Incident power

REFLECTION AND TRANSMISSION

- normal incident plane waves
- constant cross-sectional area
- change in impedance

Z_1

incident →

$$p^+ = p^+(t - x/c_1)$$

← reflected

$$p^- = p^-(t - x/c_1)$$

transmitted →

$$p^{tr} = p^{tr}(t - x/c_2)$$

Z_2

REFLECTION AND TRANSMISSION

- reflection coefficient: $$R = \frac{p^-}{p^+}$$
- transmission coefficient: $$T = \frac{p^{tr}}{p^+}$$
- ... at the interface
  - continuity of pressure: $1 + R = T$
  - continuity of mass: $1 - R = Z_1/Z_2\ T$

REFLECTION AND TRANSMISSION

- solving for R and T yields:
  - reflection coefficient:

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

  - transmission coefficient:

$$T = \frac{2Z_2}{Z_2 + Z_1}$$

Impedance jump thus causes reflection … hence less absorption!

$R + T \neq 1$!

REFLECTION AND TRANSMISSION

$$I_{ac} = \frac{1}{2} P\overline{P}\,\Re(1/\overline{Z})$$

$$\frac{1}{2} P\overline{P} = p_{rms}^2$$

- sound power reflection coefficient r

$$r = \frac{W^-}{W^+} = \frac{I^- S}{I^+ S} = \frac{(p_{rms}^-)^2}{(p_{rms}^+)^2}$$

- using $I_{ac} = p_{rms}^2\,\Re(1/\overline{Z})$

$$r = R^2$$

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REFLECTION AND TRANSMISSION

- sound power transmission coefficient τ

$$\tau = \frac{W^{tr}}{W^{+}} = \frac{(p^{tr}_{rms})^2/Z_2}{(p^{+}_{rms})^2/Z_1}$$

  - which yields:

$$\tau = T^2 \frac{Z_1}{Z_2}$$

REFLECTION AND TRANSMISSION

- in terms of impedance ...

$$r = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

- and

$$\tau = \frac{4 Z_1 Z_2}{(Z_2 + Z_1)^2}$$

conservation of energy

$$r + \tau = 1$$

REFLECTION AND TRANSMISSION

- $Z_2 >> Z_1$ (Rigid wall) yields
  - R = 1, r = 1
  - T = 2, τ = 0
  - pressure doubling

$$p_{wall} = p^+ + p^- = (1+R)p^+ = 2p^+$$

- ... all power reflected

REFLECTION AND TRANSMISSION

- $Z_2 << Z_1$ (Pressure release)
  - R = -1, r = 1
  - T = 0, $\tau = 0$
  - pressure zero

$$p_{wall} = (1 + R)p^{+} = 0$$

- ... all power reflected

REFLECTION AND TRANSMISSION

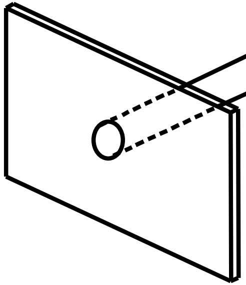
- $Z_2 = Z_1$ (Matched impedance)
  - R = 0, r = 0
  - T = 1, τ = 1
  - pressure undisturbed

$$p_{wall} = (1 + R)p^+ = p^+$$

- ... all power transmitted

Full absorption for matched impedance!

ABSORPTION OF SOUND (RESONATORS)

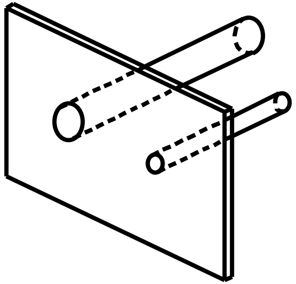


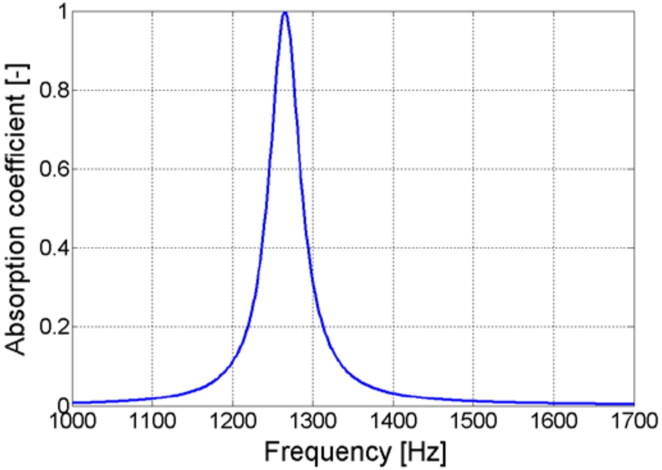
quarter-wave resonators

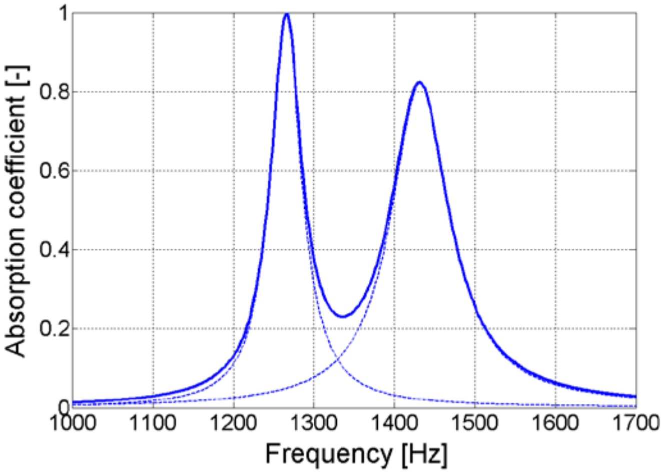
resonator = low impedance

surface = high impedance

total = matched impedance



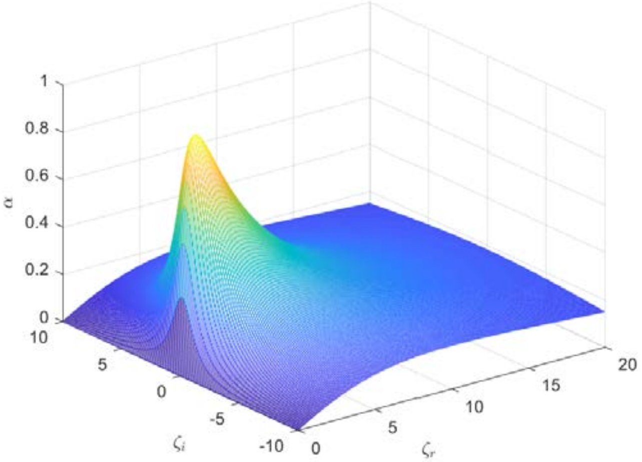


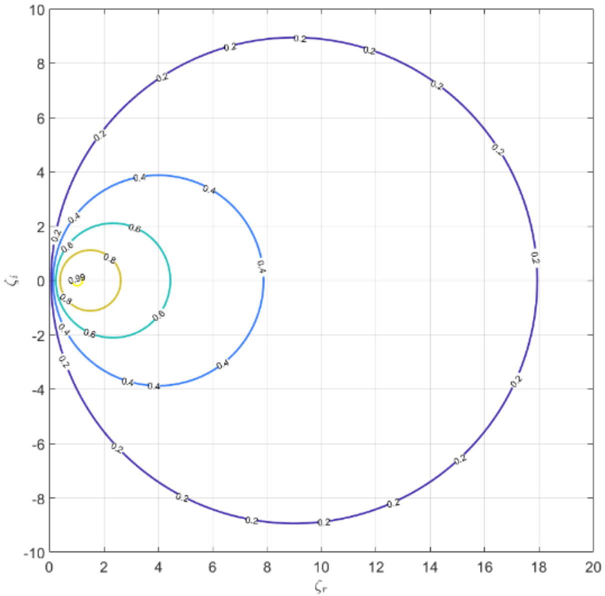


$$\zeta = \frac{Z}{\rho_0 c_0}$$

ABSORPTION OF SOUND

- Absorption v. impedance (normal incidence) $\alpha = \frac{2(\zeta + \bar{\zeta})}{(\zeta + 1)(\bar{\zeta} + 1)}$



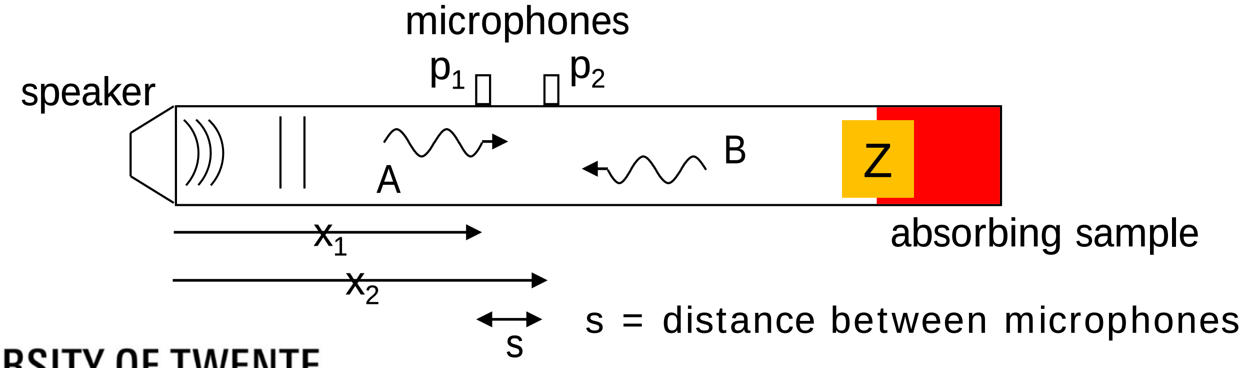


ABSORPTION OF SOUND (MEASUREMENT)

- origin coordinate system at p_1

$$p_1 = A + B$$
$$p_2 = Ae^{-iks} + Be^{iks}$$

$$\alpha \equiv \frac{W_{ac}}{W_{in}} = \frac{A\bar{A} - B\bar{B}}{A\bar{A}}$$



ABSORPTION OF SOUND (MEASUREMENT)

$$\begin{aligned} p_1 &= A + B \\ p_2 &= Ae^{-iks} + Be^{iks} \end{aligned}$$

- Non-iso standard!

$$\begin{aligned} 1 &= \tilde{A} + \tilde{B} \\ H_{21} &= \tilde{A}e^{-iks} + \tilde{B}e^{iks} \end{aligned}$$

$$\tilde{A} = \frac{A}{p_1}$$

$$\tilde{B} = \frac{B}{p_1}$$

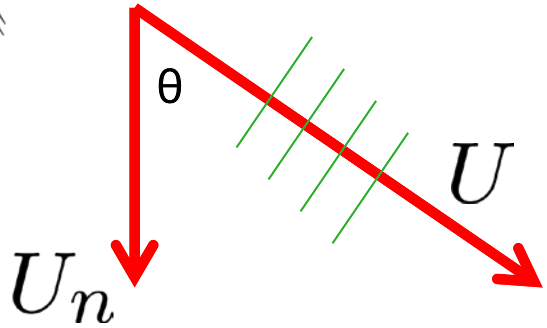
$$\begin{aligned} R &= \frac{B}{A} = \frac{\tilde{B}}{\tilde{A}} \\ &= \frac{H_{21} - e^{-iks}}{e^{iks} - H_{21}} \end{aligned}$$

$$\longrightarrow \quad \alpha = 1 - |R|^2$$

Note: independent of position!

ABSORPTION OF OBLIQUE SOUND (MEASUREMENT)

- Absorption depends does not only dependent on the impedance but also on e.g. the angle of incidence (or, more general, the incident sound field)



plane wave

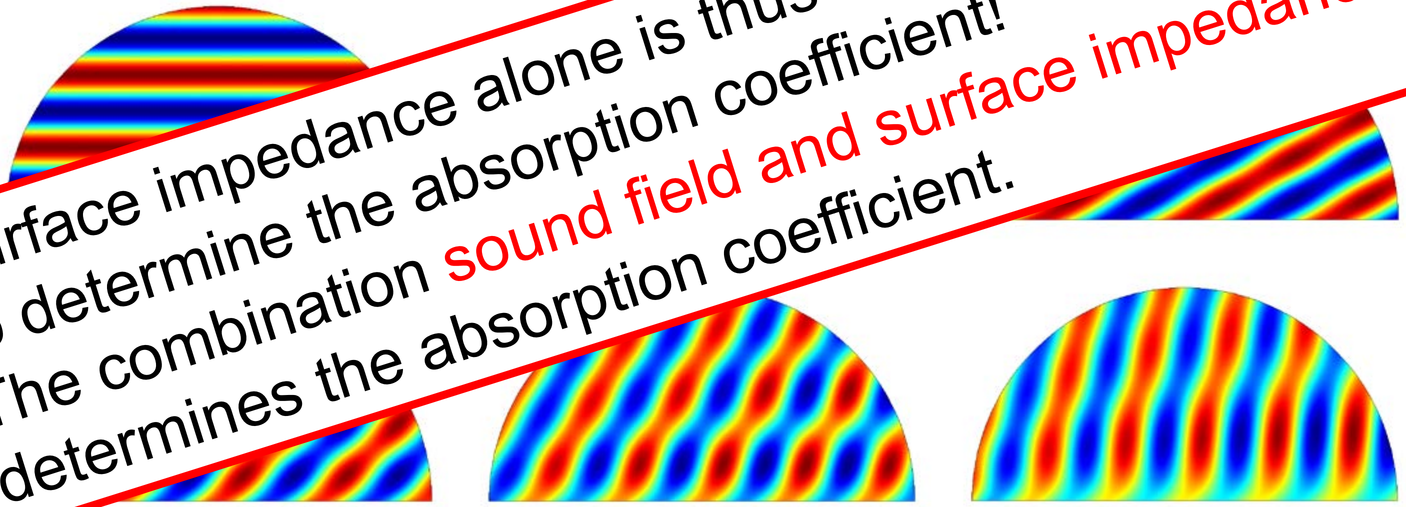
$$\frac{P}{U} = Z_0$$

normal impedance

$$Z_n = \frac{P}{U_n} = \frac{P}{U cos(\theta)} = \frac{Z_0}{cos(\theta)}$$

ABSORPTION OF SOUND

- Constant impedance ($Z_n = \rho_0 \, c_0$; $\zeta = 1 + 0i$)

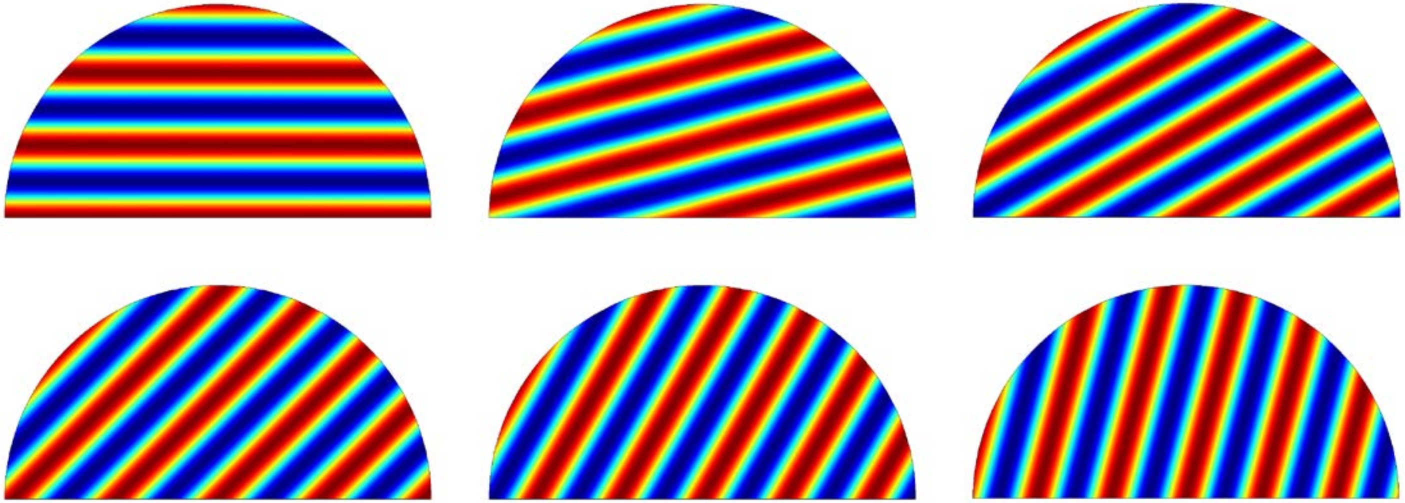


Surface impedance alone is thus not enough to determine the absorption coefficient! The combination sound field and surface impedance determines the absorption coefficient.

ABSORPTION OF OBLIQUE SOUND

$$Z_n = \frac{Z_0}{cos(\theta)}$$

$Z_n = \rho_0 c_0$ | $Z_n = 1.04 \rho_0 c_0$ | $Z_n = 1.15 \rho_0 c_0$



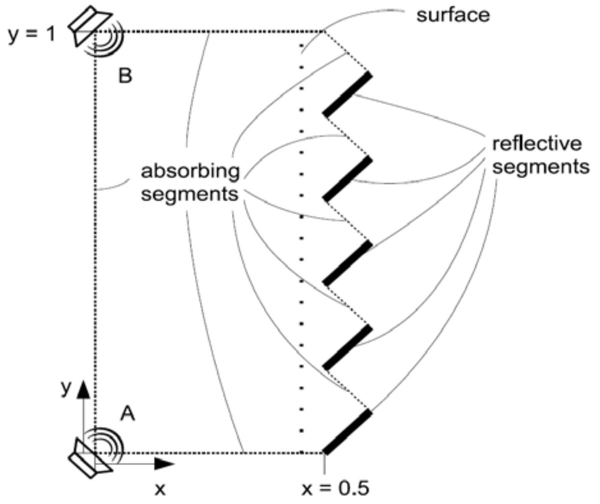
$Z_n = 1.41 \rho_0 c_0$ | $Z_n = 2 \rho_0 c_0$ | $Z_n = 3.86 \rho_0 c_0$

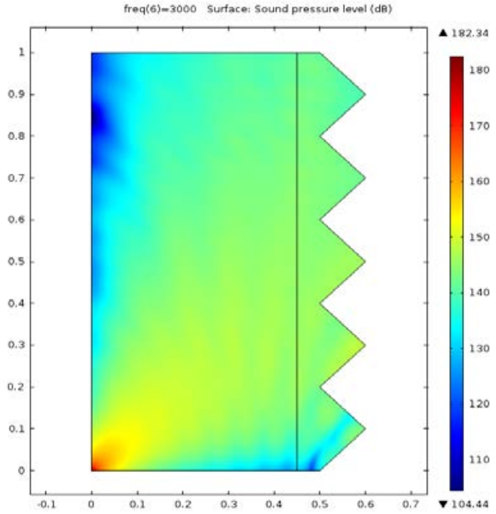
ABSORPTION OF SOUND (IN-SITU MEASUREMENT)

Louvre door

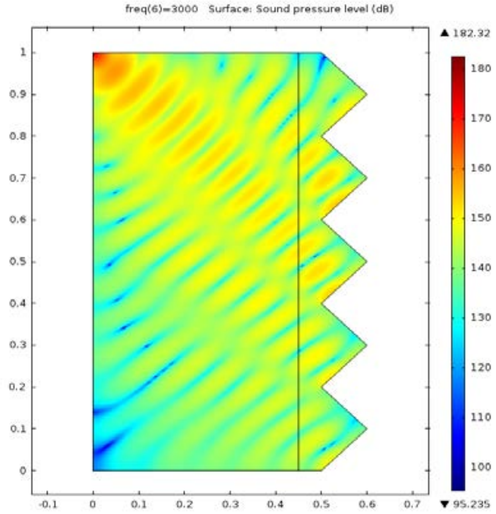
ABSORPTION OF SOUND (IN-SITU MEASUREMENT)

- Louvre door/window



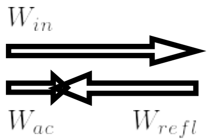


SPL Source at A



SPL Source at B

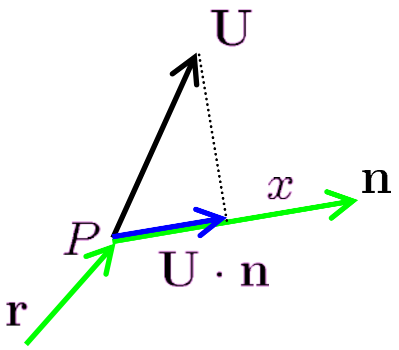
ABSORPTION OF SOUND (IN-SITU MEASUREMENT)



- LOCAL plane wave assumption for in-situ measurement of absorption
- Determine the amplitudes of the incident and reflected wave at the measurement position (like in an impedance tube)

$$
\begin{aligned}
P &= Ae^{-ikx} + Be^{ikx} = A + B \\
\mathbf{U}\cdot\mathbf{n} &= \frac{1}{\rho c}\left(Ae^{-ikx} - Be^{ikx}\right) = \frac{1}{\rho c}(A - B)
\end{aligned}
$$

$$
\begin{aligned}
A(\mathbf{r},\mathbf{n}) &= (P + \rho c\mathbf{U}\cdot\mathbf{n})/2 \\
B(\mathbf{r},\mathbf{n}) &= (P - \rho c\mathbf{U}\cdot\mathbf{n})/2
\end{aligned}
$$



ABSORPTION OF SOUND (IN-SITU MEASUREMENT)

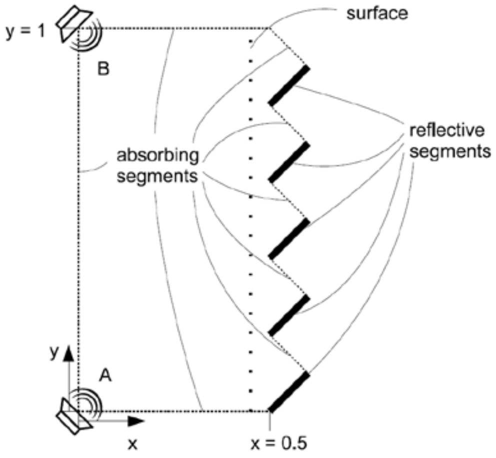
- Determine the incident intensity at every location and the associated incident power

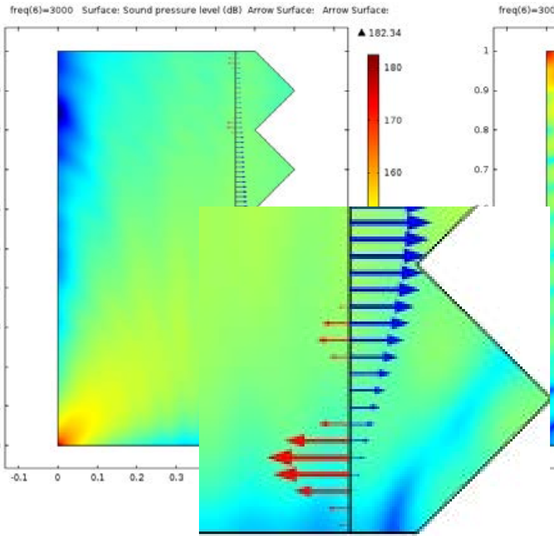
$$\begin{aligned} I_{in} &= A\overline{A}/(2\rho c) \\ I_{refl} &= B\overline{B}/(2\rho c) \end{aligned} \longrightarrow W_{in} = \int I_{in} dS \longrightarrow \alpha = \frac{W_{ac}}{W_{in}}$$

- = LPW-method

ABSORPTION OF SOUND (IN-SITU MEASUREMENT)

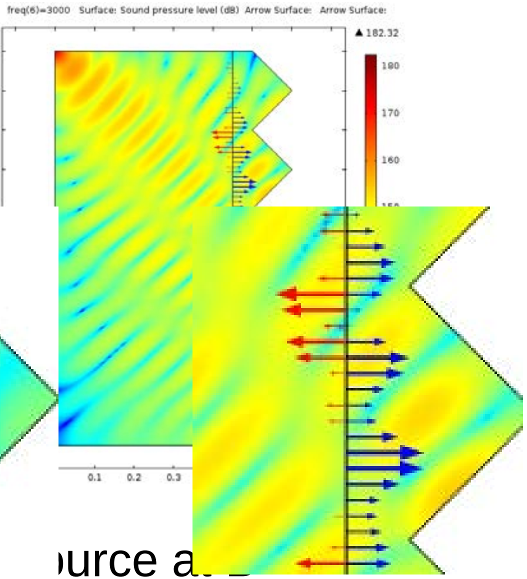
- Louvre door





Source a

SPL, I_{in} and I_{refl}



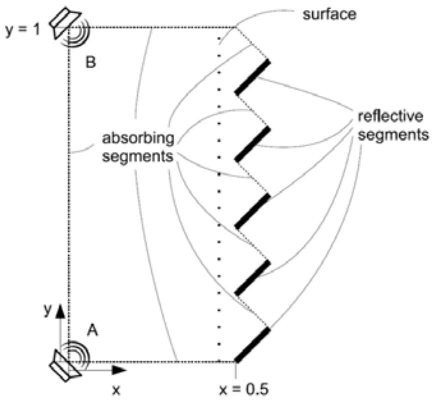
urce a

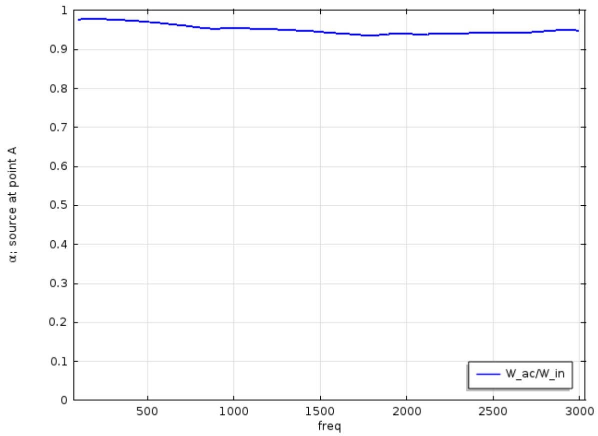
SPL, I_{in} and I_{refl}

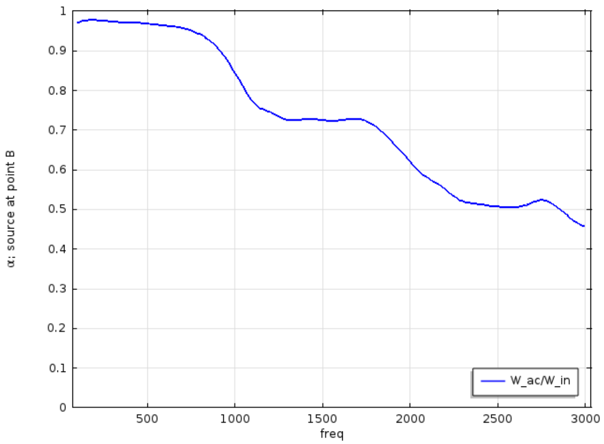
UNIVERSITY OF TWENTE.

ABSORPTION OF SOUND (IN-SITU MEASUREMENT)

- Louvre door







Source at A

Source at B

DEMONSTRATION

- Impedance tube
- LPW method